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TOPICAL ISSUE

On Combination of the Processes of Reconstruction and Guaranteeing Control[1]

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Abstract—Consideration was given to the problem of controlling a system of ordinary differential equations under incomplete information about the phase states. Given was an algorithm to solve it on the basis of a combination of the "real-time" reconstruction processes and feedback control.

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1. INTRODUCTION. FORMULATION OF THE PROBLEM

The method of control with a model was suggested by N.N. Krasovskii at the early 1970s for the finite-dimensional controlled systems with dynamic perturbations [1, 2]. It enables one to construct the feedback control laws that are stable to small noise in the channel of phase state observation. The method of control with model was used in [3] to solve some problems of control with observation of incomplete signal about the phase states. In these solutions, the auxiliary controlled system (model) configured in the control loop serves not only for immediate generation of the control actions in the original system, but also for approximate dynamic reconstruction of its full phase states, the reconstructed states being used in the stable control unit. The present paper aims at demonstrating how the method of reconstruction-control with model can be applied to the problems of game control in the case of measuring part of the coordinates of the phase vector.

Consideration is given to the problem of robust control of the system of ordinary differential equations

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), y(t), u(t), v(t)), \\ \dot{y}(t) &= f_0(t, x(t), y(t)), \quad t \in T = [t_0, \vartheta] \end{aligned} \tag{1.1}$$

with the initial condition

$$x(t_0) = x_0, \quad y(t_0) = y_0, \tag{1.2}$$

where $x \in R^N$, $y \in R^n$, $n \geqslant N$, $u \in R^m$ is the control, $v \in R^g$ is the perturbation, the functions $f_0(t,x,y) = f_1(t,y) + f_2(t,y)x$, $f(\cdot) : Z = T \times R^N \times R^n \times R^m \times R^q \to R^N$, $f_1(\cdot) : T \times R^n \to R^n$, $f_2(\cdot) : T \times R^n \to R_M^{n\times N}$ are Lipschitzian in the arguments from the spaces R^N and R^n and continuous in the rest of the arguments, and $R_M^{n\times N}$ denotes the space of $(n \times N)$ matrices with a Euclidean norm.

[1] The paper is based on the materials of the plenary report.

The considered problem can be formulated descriptively as follows. System (1.1) is subjected to the action of control $u = u(t) \in P$ generated in the course of process development and an unknown perturbation $v = v(t) \in Q$. Here, $P \subset R^m$ and $Q \subset R^g$ are bounded closed sets, the "resources" of control and perturbations, respectively. A uniform net $\Delta = \{\tau_i\}_{i=0}^{m}$, $\tau_0 = t_0$, $\tau_m = \vartheta$, $\tau_{i+1} = \tau_i + \delta$ with the step δ was selected over the time interval T. One of the phase coordinates $x(\tau_i)$ or $y(\tau_i)$ is measured (with error) at the time instants τ_i. The results of measuring the vector $\xi_i \in R^N$ or $\eta_i \in R^n$ satisfy, respectively, the inequalities

$$|\xi_i - x(\tau_i)|_N \leqslant h, \qquad |\eta_i - y(\tau_i)|_n \leqslant h, \tag{1.3}$$

where $h \in (0, 1)$ is the value of the measurement error. In what follows, $|\cdot|_N$ denotes the Euclidean norm in the space R^N, and $z(t) = (x(t), y(t))$, the system phase trajectory. It is desired to give a law for generation of feedback control of system (1.1)

$$u(t) = u^e(t) = u_i^e(\tau_i, \nu_i) \in P, \quad t \in [\tau_i, \tau_{i+1}), \quad i \in [0 : m-1]$$

($\nu_i = \xi_i$ for measurement of the component x, and $\nu_i = \eta_i$ for measurement of the component y) such that, no matter what the unknown perturbation $v = v(\cdot)$ is, at the instant $t = \vartheta$ the system phase state $(x(\cdot), y(\cdot)) = (x(\cdot; t_0, x(t_0), y(t_0), u(\cdot), v(\cdot)), y(\cdot; t_0, x(t_0), y(t_0), u(\cdot), v(\cdot)))$ gets into the sufficiently small ε-neighborhood of the given set $M \subset R^{N+n}$, that is, the set M^ε. Here and below, M^ε denotes the closure of the ε-neighborhood of the set M.

Using the terminology of the theory of positional differential games [1, 2], the choice of the control law, that is, the method of measuring the parameter $u(t)$, is in hands of some "player" that has to select the law so that to support the aforementioned property of motion under any possible realization of the action $v = v(t)$. We emphasize that the nature of the action v is indifferent and may be a program control or feedback positional control generated by somebody. Only two conditions must be satisfied: first, the realization of $v(t)$ must be a Lebesgue-measurable function over the interval T and, second, it must satisfy the inclusion $v(t) \in Q$ for almost all (a.a.) $t \in T$.

The present paper describes an algorithm to solve the above problem which is based on the method of dynamic inversion (dynamic approximation of controls) developed in [3, 4] and on the method of stable tracks known in the theory of positional control [1]. Owing to the incompleteness of information–namely, the possibility of measuring only part $x(\tau_i)$ or $y(\tau_i)$ of the system phase state at the instants τ_i rather than the entire state $(x(\tau_i), y(\tau_i))$—together with the control unit we use the additional unit of dynamic restoration of the unknown coordinate which is called in the control theory the observer. It plays the part of the provider of information about the current full system phase state. This information is sent in real time to the "control" unit generating the control u according to the feedback law.

We notice that the fundamentals of the theory of positional control were laid in [1, 2]. Yet, these publications discussed the problems of guaranteed control in the cases of measuring the entire phase state with error, that is, for "full" information about the phase trajectories. The present paper considers the problem of guaranteed hitting the given set by the phase system trajectory at measuring only "part" of the phase state (measurement of "part of the coordinates"). Therefore, we consider the game christened the approach-evasion game. As was noted on page 49 of the monograph [1], the performance functional and, consequently, in this game the game cost lacks. At the same time, as was noted in this monograph, the "approach-evasion game... defines the basis for the study of many differential games where a nontrivial functional occurs" [1, p. 50].

We assume in what follows that the function f satisfies the condition for saddle point in "small game" [1, p. 56]: for all $t \in T$, $x \in R^N$, $y \in R^n$, for each $l \in R^N$:

$$\sup_{u \in P} \inf_{v \in Q} \langle l, f(t, x, y, u, v)\rangle_N = \inf_{v \in Q} \sup_{u \in P} \langle l, f(t, x, y, u, v)\rangle_N;$$

here and below, $\langle\cdot,\cdot\rangle_N$ is the scalar product in the Euclidean space R^N. The *permissible control* is any measurable function $u(\cdot): T \mapsto P$, the *permissible perturbation* is any measurable function $v(\cdot): T \mapsto Q$. The sets of all permissible controls and perturbations are denoted, respectively, by $\mathcal{U}$ and $\mathcal{V}$. The motion of system (1.1) under the action of $u(\cdot) \in \mathcal{U}$ and $v(\cdot) \in \mathcal{V}$ is the function $(x(\cdot), y(\cdot))$, that is, the solution of system (1.1) in the Carathéodori sense. The following lemma is true.

Lemma 1. *It is possible to indicate a number d_* such that the inequality*

$$\operatorname*{vrai\,sup}_{t\in T}\left\{|x(t)|_N + |y(t)|_n + |\dot{x}(t)|_N + |\dot{y}(t)|_n\right\} \leqslant d_*$$

is valid uniformly on all $(x(\cdot), y(\cdot)) \in Z_T = ((x(\cdot|u(\cdot), v(\cdot)), y(\cdot|u(\cdot), v(\cdot))) : u(\cdot) \in \mathcal{U},\ v(\cdot) \in \mathcal{V})$.

The symbol Z^ε denotes the closed ε-neighborhood of the set Z in $R^N \times R^n$ ($\varepsilon \geqslant 0$, Z^0 is the closure of the set Z). We fix the families $(M_t)_{t\in T}$ and $(N_t)_{t\in T}$ of the closed sets in $R^N \times R^n$ such that the sets $M = \cup_{t\in T}\{t\} \times M_t$ and $N = \cup_{t\in T}\{t\} \times N_t$ are closed in $T \times R^N$ and $T \times R^n$, respectively. It is said for each $\varepsilon \in [0, +\infty)$ that the motion $(x(\cdot), y(\cdot)) : T \mapsto R^N \times R^n$ *ε-guided* if there exists $\tau \in I$ such that $(x(\tau), y(\tau)) \in M_\tau^\varepsilon$ and $(x(t), y(t)) \in N_t^\varepsilon$ for all $t \in [t_0, \tau]$. The present authors are interested in the control laws guaranteeing that for a sufficient precision of the observed signal about the phase states (1.1) is ε-guided with an arbitrarily small $\varepsilon > 0$. Therefore, it is necessary to indicate control laws providing ε-guidance of motion of system (1.1) in the case where h characterizing the error of measurement of the observed signal does not exceed h_* which depends on the number ε, that is, $h \in (0, h_*(\varepsilon))$.

2. CASE OF MEASURING THE COMPONENT $x(\cdot)$

We first consider the case of observing the component $x(t)$ and then the case of observing the component $y(t)$, the latter case being the basic one. For the purpose of constructiveness, we concentrate on the special situation corresponding to the method of stable tracks [1]. We assume that almost for all $t \in T$ the convex closed set

$$F(t, w, z) = \bigcap_{v\in Q} f(t, w, z, P, v)$$

is nonempty. Here,

$$f(t, w, z, P, v) = \mathrm{co}\{f(t, w, z, u, v) : u \in P\},$$

and co$P$ denotes the closure of the convex hull of the set P. By the *track* we mean any absolutely continuous function $(w(\cdot), z(\cdot))$, satisfying the condition $(w(t_0), z(t_0)) = (x_0, y_0)$ and solving the inclusion

$$\begin{cases} \dot{w}(t) \in F(t, w(t), z(t)) \\ \dot{z}(t) = f_0(t, w(t), z(t)). \end{cases} \tag{2.1}$$

The set of all tracks is denoted by $\mathcal{R}$. As can be easily seen, a Lebesgue-measurable function $r(\cdot) = r(\cdot; w(\cdot), z(\cdot))$ such that $\dot{w}(t) = r(t; w(t), z(t)) \in F(t, w(t), z(t))$ for a.a. $t \in T$ corresponds to each track $(w(\cdot), z(\cdot))$. The track $(w(\cdot), z(\cdot))$ will be said to be *generated* by the function $r(\cdot)$. In what follows, the track $(w(\cdot), z(\cdot))$ is referred to as 0-guided if there exists $\tau = \tau(w(\cdot), z(\cdot)) \in T$ such that $(w(\tau), z(\tau)) \in M_\tau$, $(w(t), z(t)) \in N_t$ for all $t \in [t_0, \tau]$.

We notice that the system controllability and reachability conditions are of import in the control theory at studying the problem of driving the phase trajectory of a dynamic system to the given objective set. We follow here [1] and do without the classical form of such conditions replacing them by the conditions for existence of the 0-guided track.

We follow [1] and call the control law based on the results of observation of the full phase state the *strategy* and define it as the pair (Δ, U) where $\Delta = (\tau_i)_{i=0}^m$, $\tau_0 = t_0$, $\tau_m = \vartheta$, is the *decomposition* of the decomposition of the segment T, with diameter $\delta = \tau_{i+1} - \tau_i$, and

$$U : T \times R^N \times R^n \mapsto P$$

is the feedback. We fix the model obeying the equation

$$\dot{p}(t) = f_0(t, \xi_i, p(t)) \quad \text{for a.a.} \quad t \in [\tau_i, \tau_{i+1}), \quad p(t_0) = y_0,$$

where ξ_i is the result of observing the component $x(\tau_i)$. Any sectionally continuous function $\xi(\cdot) : T \to R^N$ is called the signal x-input.

For the observation precision h ($h \geqslant 0$), the *motion* generated by the above strategy is a function given by

$$(x(\cdot), y(\cdot)) = (x(\cdot|u(\cdot), v(\cdot)), y(\cdot|u(\cdot), v(\cdot))),$$

where $v(\cdot) \in \mathcal{V}$ and the equality

$$u(t) = u_i = U(\tau_i, \xi_i, p(\tau_i))$$

is satisfied for all $i = 0, 1, \ldots, m-1$, $t \in [\tau_i, \tau_{i+1})$, under some $\xi_i \in R^N$ such that

$$|\xi_i - x(\tau_i)|_N \leqslant h.$$

The set of all such motions is denoted by $Z_h(\Delta, U)$.

The family of strategies $((\Delta_h, U_h))_{h>0}$ is said to *guarantee stable guidance* if for any $\varepsilon > 0$ there exists $h_0 > 0$ such that for each $h \in (0, h_0]$ any motion from $Z_h(\Delta_h, U_h)$ is ε-guided. Here, $\Delta_h = (\tau_{i,h})_{i=1}^{m_h}$ is the family of uniform decompositions of the segment T such that

$$\tau_{i,0} = t_0, \quad \tau_{i,m_h} = \vartheta, \quad \tau_{i+1,h} \equiv \tau_{i,h} + \delta(h), \quad \delta(h) = (\vartheta - t_0)/m_h.$$

We then fix the family $(\Delta_h)_{h>0}$ of decompositions of the segment T with property

$$\lim_{h \to 0} \delta(h) = 0$$

and the positive function $h \mapsto \zeta(h)$ of the positive argument h such that

$$\lim_{h \to 0} \zeta(h) = 0.$$

Let $\omega(\delta) = \sup\{|f(t_1, x_2, y, u, v) - f(t_2, x_1, y, u, v)|_N : \ t_1, t_2 \in T,\ |t_1 - t_2| < \delta,\ (x, y) \in Z_*\}$, $Z_* \in R^{N \times n}$ be a bounded set where all phase states of systems (1.1)—($x(t)$, $y(t)$) and (2.1)—($w(t)$, $z(t)$) remain. Condition 1 is required below.

Condition 1. *For any $t \in T$, $x \in R^N$, $y \in R^n$, valid are the relations*

$$\Phi(t, x, y) = \bigcap_{u \in P} \bigcup_{v \in Q} f(t, x, y, u, v) \neq \emptyset,$$
$$F(t, x, y) \subset \Phi(t, x, y).$$

The following theorem is a direct generalization of the main assertion [1] characterizing the method of stable tracks for the finite-dimensional systems.

Theorem 1. (1) *Let* $(w^0(\cdot), z^0(\cdot))$ *be a* 0*-guided track. Then, the strategy family* $(\Delta_h, U_h)_{h>0}$ *where* U_h *are such that*

$$\begin{gathered}\max_{v\in Q}\left\langle \xi - w^0(t), f(t,\xi,p,U_h(t,\xi,p),v)\right\rangle_N \\ \leqslant \min_{u\in P}\max_{v\in Q}\left\langle \xi - w^0(t), f(t,\xi,p,u,v)\right\rangle_N + \zeta(h) \\ \text{for} \quad t = \tau_{i,h}, \quad \xi = \xi_i, \quad p = p(\tau_i),\end{gathered} \tag{2.2}$$

guarantees stable guidance; moreover,

$$\lim_{h\to 0}\sup\left\{\left|(x_h(t),y_h(t)) - (w^0(t),z^0(t))\right|_{N+n} : \ t\in T,\ (x_h(\cdot),y_h(\cdot))\in Z_h(\Delta_h,U_h)\right\} = 0. \tag{2.3}$$

(2) *If Condition 1 is satisfied, then the family of strategies guaranteeing stable guidance exists if and only if there exists* 0*-guiding track.*

Proof. We first prove the first assertion of the theorem. By the definition of the 0-guiding track,

$$\begin{gathered}(w^0(\tau), z^0(\tau)) \in M_\tau \quad \text{for some} \quad \tau\in T,\\ (w^0(t), z^0(t)) \in N_t \quad \text{for all} \quad t\in[t_0,\tau].\end{gathered}$$

We fix $h>0$. Let $(x_h(\cdot), y_h(\cdot))$ be the motion generated by the strategy (Δ_h, U_h), that is, the solution of system (1.1) corresponding to some function $v(\cdot)\in\mathcal{V}$ and piecewise constant control $u_h(\cdot)$ given by

$$\begin{gathered}u_h(t) = u_{h,i} = U_h(\tau_i, \xi_i^h, p(\tau_i)), \quad t\in\delta_i = [\tau_i,\tau_{i+1}), \quad \tau_i = \tau_{i,h},\\ \xi_i^h \in R^N, \quad |\xi_i^h - x_h(\tau_i)|_N \leqslant h.\end{gathered} \tag{2.4}$$

Here, $p(\cdot)$ is the phase trajectory of the model, that is, $p(\cdot)$ is the solution of the equation

$$\dot p(t) = f_0(t,\xi_i^h, p(t)) \quad \text{for a.a.} \quad t\in[\tau_i,\tau_{i+1}), \quad p(t_0) = y_0.$$

We estimate the variation of

$$\lambda^h(t) = |z_h(t) - g_0(t)|^2_{N+n}, \quad t\in T$$

assuming that $z_h(\cdot) = (x_h(\cdot), y_h(\cdot))$, $\quad g_0(\cdot) = (w^0(\cdot), z^0(\cdot))$. For a.a. $t\in T$, we have

$$\dot\lambda^h(t) = \nu_h^{(1)}(t) + \nu_h^{(2)}(t),$$

where

$$\begin{gathered}\nu_h^{(1)}(t) = 2\langle \dot x_h(t) - \dot w^0(t), x_h(t) - w^0(t)\rangle_N,\\ \nu_h^{(2)}(t) = 2\langle \dot y_h(t) - \dot z^0(t), y_h(t) - z^0(t)\rangle_n \quad \text{for a.a.} \quad t\in T.\end{gathered}$$

One can readily see that

$$\nu_h^{(1)}(t) = \left\langle f(t,x_h(t),y_h(t),u_{h,i},v(t)) - r^0(t), s_{h,1}(t)\right\rangle_N, \tag{2.5}$$

$$\nu_h^{(2)}(t) = \left\langle f_0(t,x_h(t),y_h(t)) - f_0(t,w^0(t),z^0(t)), s_{h,2}(t)\right\rangle_n \quad \text{for a.a. } t\in T. \tag{2.6}$$

Here,

$$s_{h,1}(t) = x_h(t) - w^0(t),$$
$$s_{h,2}(t) = y_h(t) - z^0(t),$$

and $r^0(\cdot) = r(\cdot; w^0(\cdot), z^0(\cdot)) \in \mathcal{R}$ is the function generating the track $(w^0(\cdot), z^0(\cdot))$. With regard for Lipschitzness of the function f_0, we establish that

$$\nu_h^{(2)}(t) \leqslant L_1|s_{h,2}(t)|_n^2 + L_1|s_{h,2}(t)|_n|s_{h,1}(t)|_N \quad \text{for a.a. } t \in T. \tag{2.7}$$

Using the Gronwall lemma [5], it is easy to establish the estimate

$$|y_h(t) - p(\tau_i)|_n \leqslant k_0(\delta + h), \quad t \in \delta_i, \tag{2.8}$$

where $\delta = \delta(h)$. In virtue of (2.8), Lemma 1, and the inequality $|x_h(\tau_i) - \xi_i^h|_N \leqslant h$, we establish additionally for $t \in \delta_i$ that

$$\begin{aligned}
&\Big\langle f(t, x_h(t), y_h(t), u_{h,i}, v(t)) - r(t; w^0(t), z^0(t)), s_{h,1}(t)\Big\rangle_N \\
&\leqslant \Big\langle f(t, x_h(t), y_h(t), u_{h,i}, v(t)) - r(t; w^0(t), z^0(t)), \xi_i^h - w^0(\tau_i)\Big\rangle_N \\
&\quad + k_1\left(h + \int_{\tau_i}^{t} \Big\{|\dot{x}_h(\tau)|_N + |\dot{w}^0(\tau)|_N\Big\}\, d\tau\right) \\
&\leqslant \Big\langle f(t, x_h(t), p(\tau_i), u_{h,i}, v(t)) - r(t; w^0(t), z^0(t)), \xi_i^h - w^0(\tau_i)\Big\rangle_N + k_2(h + \delta).
\end{aligned} \tag{2.9}$$

Let the vector $v_{h,i}$ be determined from the condition

$$\begin{aligned}
&\min_{u \in P}\Big\langle \xi_i^h - w^0(\tau_i), f(\tau_i, x_h(\tau_i), p(\tau_i), u, v_{h,i})\Big\rangle_N \\
&= \max_{v \in Q}\min_{u \in P}\Big\langle \xi_i^h - w^0(\tau_i), f(\tau_i, x_h(\tau_i), p(\tau_i), u, v)\Big\rangle_N.
\end{aligned} \tag{2.10}$$

Then, the expression [1, p. 60]

$$r(t; w^0(t), z^0(t)) = \sum_{j=1}^{N+1} \alpha_t^{(j)} f(t, w^0(t), z^0(t), u_t^{(j)}, v_{h,i}), \tag{2.11}$$

$$\alpha_t^{(j)} \geqslant 0, \quad u_t^{(j)} \in P, \quad \sum_{j=1}^{N+1} \alpha_t^{(j)} = 1$$

is valid in virtue of the well-known Carathéodori theorem and the inclusion

$$r(t; w^0(t), z^0(t)) \in f(t, w^0(t), z^0(t), P, v_{h,i}).$$

At that, the relations

$$\begin{aligned}
&\left|r(t; w^0(t), z^0(t)) - \sum_{j=1}^{N+1} \alpha_t^{(j)} f(\tau_i, w^0(\tau_i), z^0(\tau_i), u_t^{(j)}, v_{h,i})\right|_N \leqslant \varrho(\delta) \;\text{ for }\; t \in \delta_i, \\
&\Big|r(t; w^0(t), z^0(t)) - \varrho_0(\tau_i, x_h(\tau_i), y_h(\tau_i))\Big| \\
&\leqslant k_3\left\{\Big|w^0(t) - x_h(t)\Big|_N + \Big|z^0(t) - y_h(t)\Big|_n\right\} + \varrho(\delta),
\end{aligned} \tag{2.12}$$

where

$$\varrho(\delta) \to 0 \quad \text{for} \quad \delta \to 0,$$

$$\varrho_0(\tau_i, x_h(\tau_i), y_h(\tau_i)) = \sum_{j=1}^{N+1} \alpha_t^{(j)} f\left(\tau_i, x_h(\tau_i), y_h(\tau_i), u_t^{(j)}, v_{h,i}\right) \tag{2.13}$$

are valid. The estimates ($i \in [0 : m_h - 1]$)

$$\begin{aligned}
&\left\langle f(t, x_h(t), y_h(t), u_{h,i}, v(t)) - r(t; w^0(t), z^0(t)), s_{h,1}(t) \right\rangle_N \\
&\leqslant \left\langle \xi_i^h - w^0(\tau_i), f(\tau_i, x_h(\tau_i), p(\tau_i), u_{h,i}, v(t)) \right\rangle_N \\
&- \sum_{j=1}^{N+1} \left\langle \xi_i^h - w^0(\tau_i), \alpha_t^{(j)} f(\tau_i, x_h(\tau_i), p(\tau_i), u_t^{(j)}, v_{h,i}) \right\rangle \\
&+ k_4 \left(h + \delta + |x_h(t) - w^0(t)|_N\right) \\
&\times \left\{ \left|w^0(t) - x_h(t)\right|_N + \left|z^0(t) - y_h(t)\right|_n + h + \omega(\delta) + \delta \right\} + k_5 \varrho(\delta)
\end{aligned} \tag{2.14}$$

follow from (2.9)–(2.11). Taking into consideration the rules for determination of the controls $u_{h,i}$ (see (2.4)), as well as v_i^h (see (2.10)), we deduce from (2.14) along the same lines as in [1, p. 61] that

$$\begin{aligned}
&\left\langle f(t, x_h(t), y_h(t), u_{h,i}, v(t)) - r(t; w^0(t), z^0(t)), s_{h,1}(t) \right\rangle_N \\
&\leqslant k_6 \left\{ \left|w^0(t) - x_h(t)\right|_N^2 + \left|z^0(t) - y_h(t)\right|_n^2 + h + \omega(\delta) + \delta + \varrho(\delta) + \zeta(h) \right\}.
\end{aligned} \tag{2.15}$$

From (2.5), (2.7), and (2.15), we determine for a.a. $t \in T$ that

$$\dot{\lambda}^h(t) \leqslant k_7 \lambda^h(t) + k_8\{\zeta(h) + \varrho(\delta) + h + \omega(\delta) + \delta\},$$

whence it follows in virtue of Lemma 2.2 [6, p. 151] that

$$\lambda^h(t) \leqslant k_8 \left\{ \zeta(h) + \varrho(\delta) + h + \omega(\delta) + \delta + k_7 \int_{t_0}^{t} (t - \tau)(\zeta(h) + \varrho(\delta) + h + \omega(\delta) + \delta) d\tau \right\}.$$

Thus,

$$\lambda^h(t) \leqslant k_9(\zeta(h) + \varrho(\delta) + h + \omega(\delta) + \delta), \quad t \in T.$$

The constant k_{10} is put down in the explicit form. It follows from the last inequality that the family of strategies where U_h satisfies (2.2) guaranteed stable guidance. Relation (2.3) also follows from this fact because the constants k_j, $j \in [1 : 9]$ are independent of $x_h(\cdot), y_h(\cdot), w^0(\cdot), z^0(\cdot)$. Therefore, the first assertion of Theorem 1 is proved.

Now we proceed to the second assertion. Assuming the contrary, we conclude that there exists a family of strategies guaranteeing stable guidance but no 0-guiding track. This means that the motion (track) $(w_r(\cdot), z_r(\cdot))$ generated by any function $r(\cdot) \in \mathcal{R}$ either is

$$(w_r(\tau), z_r(\tau)) \notin M_\tau \quad \text{for all} \quad \tau \in T, \tag{2.16}$$

or

$$(w_r(t_r), z_r(t_r)) \notin N_{t_r} \quad \text{for some} \quad t_r \in T. \tag{2.17}$$

Since there exists a family U_h, $h \in (0,1)$, of strategies guaranteeing stable guidance, for any sequence $\varepsilon_j \to 0+$ there exists for $j \to \infty$ a sequence $h_j \to 0+$ such that any motion from the set $Z_{h_j}(\Delta_{h_j}, U_{h_j})$ is ε_j-guided. Consequently, for the sequence of functions

$$(x_j(\cdot), y_j(\cdot)) = \Big(x(\cdot|u_j(\cdot), v_j(\cdot)), y(\cdot|u_j(\cdot), v_j(\cdot))\Big) \in Z_{h_j}(\Delta_{h_j}, U_{h_j}),$$

where

$$v_j(\cdot) \in \mathcal{V}, \quad u_j(t) = u_{ji} = U_{h_j}(\tau_i, \xi_i^{(j)}, p^{(j)}(\tau_i)), \quad t \in [\tau_i, \tau_{i+1}),$$

$p^{(j)}(\cdot)$ is the solution of system

$$\dot{p}(t) = f_0(t, \xi_i^{(j)}, p(t)) \quad \text{for} \quad t \in [\tau_i, \tau_{i+1}), \quad p(t_0) = y_0,$$
$$|\xi_i^{(j)} - x_j(\tau_i)|_N \leqslant h_j,$$

the following conditions are met: there are instants $\tau_j \in T$ such that

$$(x_j(\tau_j), y_j(\tau_j)) \in M_{\tau_j}^{\varepsilon_j}, \tag{2.18}$$
$$(x_j(\tau), y_j(\tau)) \in N_\tau^{\varepsilon_j} \quad \forall \tau \in [t_0, \tau_j]. \tag{2.19}$$

We notice that the functions $(x_j(\cdot), y_j(\cdot))$ satisfy the identities

$$\dot{x}_j(t) = f(t, x_j(t), y_j(t), u_j(t), v_j(t)),$$
$$\dot{y}_j(t) = f_0(t, x_j(t), y_j(t)) \quad \text{for a.a.} \quad t \in T.$$

According to Condition 1, there are functions $v_{ji}(t, u_{ji})$ with the properties

(a) $\mu_j(t) = f(t, x_j(t), y_j(t), u_{ji}, v_{ji}(t, u_{ji})) \in F(t, x_j(t), y_j(t))$ for a.a. $t \in [\tau_i, \tau_{i+1}]$,

(b) the functions $\mu_j(t)$ are measurable.

By virtue of Lemma 1, nonemptiness, convexity, and closedness of $F(t, x_j(t), y_j(t))$ $\forall t \in T$, we can assume without loss of generality that

$$\tau_j \to \tau_* \in T,$$
$$\mu_j(\cdot) \to \mu(\cdot) \quad \text{weakly in} \quad L_2(T; R^N) \quad \text{for} \quad j \to \infty,$$
$$\mu(t) \in F(t, x_j(t), y_j(t)) \quad \text{for a.a.} \quad t \in T,$$
$$(x_j(\cdot), y_j(\cdot)) \to (w_\mu(\cdot), z_\mu(\cdot)) \quad \text{in} \quad C(T; R^N \times R^n), \tag{2.20}$$

where $(w_\mu(\cdot), z_\mu(\cdot))$ is the track generated by the function $\mu(\cdot)$. In virtue of closedness of the sets M and N from (2.18), (2.19), it follows that

$$(w_\mu(\tau_*), z_\mu(\tau_*)) \in M_{\tau_*}, \tag{2.21}$$
$$(w_\mu(\tau), z_\mu(\tau)) \in N_\tau \quad \forall \tau \in [t_0, \tau_*].$$

Indeed, by assuming the contrary we conclude that either

$$(w_\mu(\tau_*), z_\mu(\tau_*)) \notin M_{\tau_*} \tag{2.22}$$

or there exists $\tau_0 \in [t_0, \tau_*]$ such that

$$(w_\mu(\tau_0), z_\mu(\tau_0)) \notin N_{\tau_0}. \tag{2.23}$$

However, (2.22) is equivalent to

$$P_* = (\tau_*, u_\mu(\tau_*), z_\mu(\tau_*)) \notin M.$$

Since M is a closed set, there exists a neighborhood $\mathcal{U}(P_*)$ of the point P_* such that

$$\mathcal{U}(P_*) \cap M \neq \emptyset,$$

which contradicts (2.18) and (2.20). If (2.23) is true, in a similar manner we encounter a contradiction with (2.19), (2.20). Therefore, (2.21) is established. However, this contradicts (2.16), (2.17), which proves the second assertion of Theorem 1.

3. CASE OF OBSERVING THE COMPONENT $y(\cdot)$

Passing to the control laws for observation of the component $y(t)$ of the state $(x(t), y(t))$, we immediately define them as the procedures of control with model and call them below the y-procedures of control. We fix two models. The first model with dynamics (2.1), that is,

$$\begin{cases} \dot{w}(t) \in F(t, w(t), z(t)) \\ \dot{z}(t) = f_0(t, w(t), z(t)). \end{cases} \tag{3.1}$$

The dynamics of the second model obeys the system

$$\dot{w}^h(t) = f_1(t, \eta(t)) + f_2(t, \eta(t))s(t), \quad w^h(t_0) = y_0, \tag{3.2}$$

where $\eta(t)$ is the result of observation of the component $y(t)$ and $s(t) \in R^N$ is the control action. Any piecewise constant function $\eta(\cdot) : T \mapsto R^n$ is called the *signal input of the model*, and any measurable and bounded function $s(\cdot) : T \mapsto R^N$, the *control s-input of the model*. By the *model motion* generated by the signal input $\eta(\cdot)$ and the control s-input $s(\cdot)$ is meant the function $(w(\cdot), z(\cdot), w^h(\cdot))$. This motion which exists and is unique is denoted as follows:

$$\left(w(\cdot|\eta(\cdot), s(\cdot)), z(\cdot|\eta(\cdot), s(\cdot)), w^h(\cdot|\eta(\cdot), s(\cdot))\right).$$

The *y-control procedure* is defined by the triple (Δ, S, U), where

$$\begin{aligned} \Delta = (\tau_i)_{i=1}^m&\text{—decomposition of the segment } T, \\ S : (t, \eta, w^h) \mapsto S(t, \eta, w^h) : T \times R^n \times R^n \mapsto R^N&\text{—\textit{model feedback},} \\ U : (t, s, w) \mapsto U(t, s, w) : T \times R^N \times R^N \mapsto P&\text{—\textit{system feedback}.} \end{aligned}$$

The *extended motion* generated by the above y-procedure of control (Δ, S, U) under the observation precision h is the function $(x(\cdot), y(\cdot), w(\cdot), z(\cdot), w^h(\cdot))$, where

$$\begin{gathered} (x(\cdot), y(\cdot)) = (x(\cdot|u(\cdot), v(\cdot)), y(\cdot|u(\cdot), v(\cdot))), \quad v(\cdot) \in \mathcal{V}, \\ (w(\cdot), z(\cdot)) = (w(\cdot|\eta(\cdot), s(\cdot)), z(\cdot|\eta(\cdot), s(\cdot))), \\ w^h(\cdot) = w^h(\cdot|\eta(\cdot), s(\cdot)) \end{gathered}$$

and for all $i = 0, 1, \ldots, m-1$ and $t \in [\tau_i, \tau_{i+1})$

$$\begin{gathered} \eta(t) = \eta(\tau_i), \quad |\eta(\tau_i) - y(\tau_i)|_n \leqslant h, \\ s(t) = S(\tau_i, \eta(\tau_i), w^h(\tau_i)), \\ u(t) = U(\tau_i, s(\tau_i), w(\tau_i)); \end{gathered} \tag{3.3}$$

at that, the functions $\eta(\cdot)$ and $s(\cdot)$ are called, respectively, the *realizations* of the model signal input and the model control s-input under the extended motion $(x(\cdot), y(\cdot), z(\cdot), w(\cdot), w^h(\cdot))$; the function $u(\cdot)$ is called the *realization* of control in the system under the extended motion $(x(\cdot), y(\cdot), z(\cdot), w(\cdot), w^h(\cdot))$, the function $(x(\cdot), y(\cdot))$, the *motion* (of system (1.1)) generated by the control y-procedure (Δ, S, U) for the observation precision h. The set of all last motions is denoted by $Z_h(\Delta, S, U)$. The family $(\Delta_h, S_h, U_h)_{h>0}$ of y-procedures of control is said to *guarantee stable guidance* if for any $\varepsilon > 0$ there exists $h_0 > 0$ such that for each $h \in (0, h_0]$ any motion from $Z_h(\Delta_h, S_h, U_h)$ is ε-guided. We also assume that Condition 2 is satisfied.

Condition 2. $\operatorname{rank} f_2(t, y) = N \ \forall y \in R^n$.

As was noted in [1, p. 96], Condition 2 and the inequality $n \geqslant N$ provide solvability of the first equation of system (1.1), that is, the equation

$$\dot{x}(t) = f_1(t, y(t)) + f_2(t, y(t))x(t),$$

in $x(t)$, namely,

$$x(t) = f_2^+(t, y(t))(\dot{x}(t) - f_1(t, y(t))) \quad \text{for a.a.} \quad t \in T,$$

where $f_2^+(t, y)$ is the matrix pseudoinverse to $f_2(t, y)$.

The criterion for existence of the family of y-procedures of control guaranteeing stable guidance is the same as for the family of strategies U_h (see assertion (2) of Theorem 1). The fact that the model feedbacks S_h may be selected so that the model input $s(t)$ arbitrarily correctly (in the root mean square) reconstructs the unobservable component $x(t)$ of system state under sufficient precision of observations plays the key role in substantiation of this result which is pivotal for the present note (see the final Theorem 2).

To give an exact formulation, we fix a number $\rho > 0$ such that

$$|x(t|u(\cdot), v(\cdot))|_N \leqslant \rho$$

for all $u(\cdot) \in \mathcal{U}$, $v(\cdot) \in \mathcal{V}$, $t \in T$ (such ρ, obviously, exists) and the scalar function $h \to \alpha(h) \in (0, 1)$.

Lemma 2. *Let Condition* 2 *be satisfied, and the family* $(\Delta_h, S_h, U_h)_{h>0}$ *of* y*-procedures of control be such that*

$$S_h(t, \eta, w^h) = \operatorname{argmin} \left\{ \left\langle w^h - \eta, f_2(t, \eta)\nu \right\rangle_n + \alpha(h)|\nu|_N : \nu \in R^N, \ |\nu|_N \leqslant \rho \right\} \tag{3.4}$$

for $t = \tau_i \equiv \tau_{i,h}$; U_h *be an arbitrary (possibly, multivalued) map of the Cartesian product* $T \times R^N \times R^N$ *into* P.

Then, there exists a number $K > 0$ *such that for any* $h > 0$ *and the extended motion* $(x(\cdot), y(\cdot), w(\cdot), z(\cdot), w^h(\cdot))$ *generated by the* y*-procedure of control* (Δ_h, S_h, U_h) *under the observation precision* h *valid is the estimate*

$$|s(\cdot) - x(\cdot)|^2_{L_2(T;R^N)} \leqslant K\Big[(h + \delta(h) + \omega_2(\delta) + \alpha(h))^{1/2} + (h + \delta(h) + \omega_2(\delta))\alpha^{-1}(h)\Big],$$

where $s(\cdot)$ *is a realization of the* s*-input of the model under the extended motion* $(x(\cdot), y(\cdot), w(\cdot), z(\cdot), w^h(\cdot))$. *Here,* $\omega_2(\delta) = \sup\{\|f_2(t_1, y) - f_2(t_2, y)\| : t_1, t_2 \in T, |t_1 - t_2| \leqslant \delta, \ y \in Y\}$, *the symbol* $\|\cdot\|$ *denotes the Euclidean matrix norm, and* $Y \subset R^n$ *is the bounded set where all* $\eta(t)$ *remain.*

Proof. We fix $h \in (0, 1)$, verify the last inequality, and obtain from (1.1) and (3.1) that

$$\dot{\mu}(t) = f_1(t, \eta(t)) - f_1(t, y(t)) + f_2(t, \eta(t))s(t) - f_2(t, y(t))x(t) \text{ for a.a. } t \in T,$$

where

$$\mu(t) = w^h(t) - y(t) \ \text{ for } t \in T, \quad \eta(t) = \eta(\tau_i) \ \text{ for } t \in [\tau_i, \tau_{i+1}).$$

Consequently, in virtue of the condition for Lipschitzness of the functions f_1 and f_2 in the second arguments for a.a. $t \in T$, valid is the inequality

$$\begin{aligned}\langle \dot{\mu}(t), \mu(t)\rangle_n \leqslant L|\eta(t) - y(t)|_n|\mu(t)|_n \\ + L|\eta(t) - y(t)|_n|x(t)|_N + \langle f_2(t, \eta(t))(s(t) - x(t)), \mu(t)\rangle_n.\end{aligned} \tag{3.5}$$

We note that

$$|\eta(t) - y(t)|_n \leqslant c_1(h + \delta), \quad \delta = \delta(h), \tag{3.6}$$

$$|(f_2(t, \eta(t)) - f_2(\tau_i, \eta(\tau_i))x|_n \leqslant c_2(\delta + \omega_2(\delta))|x|_N \quad \forall x \in R^N. \tag{3.7}$$

Therefore, with regard for (3.6) and (3.7) we obtain from (3.5) that

$$\begin{aligned}\frac{1}{2}\frac{d\varepsilon_h(t)}{dt} \equiv c_3(h + \delta + \omega_2(\delta)) + \langle f_2(\tau_i, \eta(\tau_i))(s(\tau_i) - x(t)), \mu(\tau_i)\rangle_n \\ + 0.5\alpha(h)\left\{\left|s(t)\right|_N^2 - \left|x(t)\right|_N^2\right\} \quad \text{for a.a. } t \in T,\end{aligned} \tag{3.8}$$

where

$$\varepsilon_h(t) = \left|\mu(t)\right|_n^2 + \alpha\int\limits_{t_0}^{t}\left\{\left|s(\tau)\right|_N^2 - \left|x(\tau)\right|_N^2\right\} d\tau.$$

In view of (3.4), for $t \in [\tau_i, \tau_{i+1})$ we establish from (3.8) that

$$\varepsilon_h(t) \leqslant \varepsilon_h(\tau_i) + c_4(h + \delta + \omega_2(\delta)). \tag{3.9}$$

The inequalities

$$\sup_{t\in I}\left|w^h(t) - y(t)\right|_n^2 \leqslant K_0(h + \delta + \alpha + \omega_2(\delta)),$$

$$\int\limits_{t_0}^{t}\left|s(\tau)\right|_N^2 d\tau \leqslant \int\limits_{t_0}^{t}\left|x(\tau)\right|_N^2 d\tau + K_1\frac{h + \delta + \omega_2(\delta)}{\alpha}$$

follow from (3.9). Condition 2 being satisfied, we establish in the standard way the lower estimate (see, for example, [4]), which proves Lemma 2.

As was noted in Section 1, solution of the control problem under consideration needs an observer enabling one to restore the nonmeasurable coordinates $x(\cdot)$. One of the variants of constructing such observer consisting of the pair model (3.2) and feedback (3.4) was given above. Assuming that Condition 3 is satisfied, we describe the order of actions to be executed to restore $x(\cdot)$, that is, the procedure of observer's operation (unit of dynamic restoration).

We fix $h \in (0, 1)$ and together with it the number $\alpha = \alpha(h)$ and the decomposition $\Delta_h = (\tau_{i,h})_{i=0}^{m_h}$. Let the control u and the perturbation v in system (1.1) be generated according to certain rules such as, for example, the feedback laws. The observer operation is decomposed into $m_h - 1$ identical steps. During the ith step executed over the interval $\delta_i = [\tau_i, \tau_{i+1})$, $\tau_i = \tau_{i,h}$, we carry out the following actions. With the knowledge of the vector $\eta(\tau_i)$ ($|\eta(\tau_i) - y(\tau_i)|_n \leqslant h$) and the model state $w^h(\tau_i)$, at the instant τ_i we calculate the following vector according to the feedback S_h of the form (3.4):

$$s_i = \arg\min\left\{\langle w^h(\tau_i) - \eta(\tau_i), f_2(\tau_i, \eta(\tau_i))\nu\rangle_n + \alpha|\nu|_N : \nu \in \mathbb{R}^N, |\nu|_N \leqslant \varrho\right\}.$$

Then, for $t \in \delta_i$ we feed to the input of model (3.2) the control

$$s(t) = s_i, \quad t \in \delta_i.$$

Under the action of this control, the model phase trajectory $w^h(t)$, $t \in \delta_i$, is generated. At the next, $(i+1)$st step, similar actions are repeated. It follows from Lemma 2 that the control $s(\cdot)$ constructed according to the above rule can serve as the rms approximation of the coordinate $x(\cdot)$.

We introduce Condition 3.

Condition 3. *The decomposition family* Δ_h *and the function* $\alpha(h)$ *are such that*

$$\delta(h) \to 0, \quad \alpha(h) \to 0, \quad (h + \delta(h) + \omega_2(\delta(h)))\alpha^{-1}(h) \to 0 \quad \text{for} \quad h \to 0.$$

Availability of the approximation $s(t)$ of the unobservable component $x(t)$ allows one to make use of the feedbacks relying on the approximate information about the full state of system (1.1). In particular, the modified feedbacks (2.2) support track approximation similar to (2.3). Namely, the following lemma is true.

Lemma 3. *Let Conditions 1–3 be satisfied, the track* $(w^0(\cdot), z^0(\cdot))$ *be generated by the function* $r^0(\cdot) = r(\cdot; w^0(\cdot), z^0(\cdot)) \in \mathcal{R}$, *and the family* $(\Delta_h, S_h, U_h)_{h>0}$ *of* y*-procedures of control obeys the conditions:* S_h *is determined according to* (3.4) *and the maps* U_h *are such that*

$$\begin{aligned} &\max_{v\in Q} \left\langle S_h(t,\eta,w^h) - w^0(t), f(t, S_h(t,\eta,w^h), \eta, U_h(t, S_h, \eta), v) \right\rangle_N \\ &\leqslant \min_{u\in P} \max_{v\in Q} \left\langle S_h(t,\eta,w^h) - w^0(t), f(t, S_h(t,\eta,w^h), \eta, u, v) \right\rangle_N + \zeta(h). \end{aligned} \tag{3.10}$$

Then,

$$\lim_{h\to 0} \max \left\{ \left| (x(t), y(t)) - (w^0(t), z^0(t)) \right|_{N+n} : t \in T,\ (x(\cdot), y(\cdot)) \in Z_h(\Delta_h, S_h, U_h) \right\} = 0.$$

Lemma 3 and Theorem 1 give rise to the main assertion of the present paper.

Theorem 2. *Let Conditions 1–3 be satisfied. Then,*

(1) *The following assertions are equivalent:*

(i) *there exists a family of* y*-procedures of control guaranteeing stable guidance,*

(ii) *there exists a family of strategies guaranteeing stable guidance,*

(iii) *there exists a* 0*-guiding track.*

(2) *Let* $(w^0(\cdot), z^0(\cdot))$ *be a* 0*-guiding track and* $r^0(\cdot) = r(\cdot; w^0(\cdot), z^0(\cdot)) \in \mathcal{R}$, *its generating function. Then, stable guidance is guaranteed by the family* $(\Delta_h, S_h, U_h)_{h>0}$ *of* y*-procedures of control where* S_h *are determined according to* (3.4) *and* U_h *satisfy* (3.10).

Proof. Verification of the second assertion suffices to prove Theorem 2. Let $(w^0(\cdot), z^0(\cdot))$ be a 0-guiding track and $r^0(\cdot)$, its generating function. We estimate variation of

$$\lambda_h(t) = |p^h(t) - p_*(t)|^2_{N+n}.$$

Here, $p^h(\cdot) = (x^h(\cdot), y^h(\cdot)) = (x(\cdot|u^h(\cdot), v(\cdot)), y(\cdot|u^h(\cdot), v(\cdot)))$ is the motion of system (1.1) generated by the y-procedure of control (Δ_h, S_h, U_h), where S_h is determined according to (3.4), and U_h, according to (3.10), $p_*(\cdot) = (w^0(\cdot), z^0(\cdot))$. We get

$$\lambda_h(t) \leqslant \lambda_h^{(1)}(t) + \lambda_h^{(2)}(t),$$
$$\lambda_h^{(1)}(t) = 2|x^h(t) - w^0(t)|^2_N, \quad \lambda_h^{(2)}(t) = 2|y^h(t) - z^0(t)|^2_n.$$

Therefore,

$$\begin{gathered} v(\cdot) \in \mathcal{V}, \quad s^h(t) = s^h(\tau_i) = S_h(\tau_i, \eta(\tau_i), w^h(\tau_i)), \\ u^h(t) = u_i^h = U_h(\tau_i, s^h(\tau_i), \eta(\tau_i)), \\ \dot{w}^h(t) = f_1(t, \eta(\tau_i)) + f_2(t, \eta(\tau_i)) s^h(\tau_i) \quad \text{for a.a.} \quad t \in \delta_i = [\tau_i, \tau_{i+1}), \\ |\eta(\tau_i) - y^h(\tau_i)|_n \leqslant h, \quad i \in [0 : m_{h-1}], \quad \tau_i = \tau_{i,h}. \end{gathered} \tag{3.11}$$

We obtain similar to (3.8) that

$$0.5\dot{\lambda}_h^{(1)}(t) = 2\Big\langle \dot{x}^h(t) - \dot{w}^0(t), x^h(t) - w^0(t) \Big\rangle_N = I_i \text{ for a.a. } t \in \delta_i, \tag{3.12}$$

where

$$I_i = 2\Big\langle f(t, x^h(t), y^h(t), u^h(t), v(t)) - r^0(t), x(t) - w^0(t) \Big\rangle_N.$$

Taking into consideration the rule for determination of the function $r^0(\cdot)$, we conclude that the representation (2.11) and inequality (2.12), where ϱ_0 are determined from (2.13), are valid. At that, x^h and y^h stand, respectively, in (2.12) and (2.13) for x_h and y_h. The estimates ($t \in \delta_i$)

$$\begin{aligned} I_i &= 2\Big\langle f(t, x^h(t), y^h(t), u^h(t), v(t)) - r(t; w^0(t), z^0(t)), x^h(t) - w^0(t) \Big\rangle_N \\ &\leqslant k_0 \left\{ \left|x^h(t) - w^0(t)\right|_N + \left|y^h(t) - z^0(t)\right|_n^2 \right\} + k_1(h + \delta + \omega(\delta)) + k_2 \left|s^h(\tau_i) - x^h(t)\right|_N \\ &+ 2\sum_{j=1}^{N+1} \Big\langle f(\tau_i, s^h(\tau_i), \eta(\tau_i), u_i^h, v(t)) - \alpha_t^{(j)} f(\tau_i, s^h(\tau_i), \eta(\tau_i), u_t^{(j)}, v_i^h), s(\tau_i) - w^0(\tau_i) \Big\rangle_N \\ &\leqslant k_0 \left\{ \left|x^h(t) - w^0(t)\right|_N + \left|y^h(t) - z^0(t)\right|_n^2 \right\} + k_3 \left(\delta + \omega(\delta) + h + \left|s^h(\tau_i) - x^h(t)\right|_N \right) \end{aligned} \tag{3.13}$$

follow from Lemma 1 and the rule for determination of the control $u^h(t)$ (see (3.11)). Here, the vector v_i^h is determined from the condition (see (2.10)):

$$\begin{aligned} &\min_{u \in P} \Big\langle s^h(\tau_i) - w^0(\tau_i), f(\tau_i, s^h(\tau_i), \eta(\tau_i), u, v_i^h) \Big\rangle_N \\ &= \max_{v \in Q} \min_{u \in P} \Big\langle s^h(\tau_i) - w^0(\tau_i), f(\tau_i, s^h(\tau_i), \eta(\tau_i), u, v) \Big\rangle_N. \end{aligned}$$

We obtain from (3.13) that

$$\begin{aligned} \lambda_h^{(1)}(t) \leqslant k_4 \Bigg(&\delta + h + \omega(\delta) + \int_{t_0}^{t} \left|s^h(\tau) - x^h(\tau)\right|_N d\tau \\ &+ \int_{t_0}^{t} \left\{ \left|x^h(\tau) - w^0(\tau)\right|_N^2 + \left|y^h(t) - z^0(t)\right|_n^2 \right\} d\tau \Bigg). \end{aligned} \tag{3.14}$$

Additionally, we have

$$\begin{gathered} 0.5\dot{\lambda}_h^{(2)}(t) = 2\Big\langle \dot{y}^h(t) - \dot{z}^0(t), y^h(t) - z^0(t) \Big\rangle_n \\ = 2\Big\langle y^h(t) - z^0(t), f_1(t, y^h(t)) - f_1(t, z^0(t)) + f_2(t, y^h(t)) x^h(t) - f_2(t, z^0(t)) w^0(t) \Big\rangle_n. \end{gathered}$$

With regard for the Lipschitzness condition, we deduce from the last equality that

$$\lambda_h^{(2)}(t) \leqslant k_4 \int_{t_0}^{t} \left\{ \left|y^h(\tau) - z^0(\tau)\right|_n^2 + \left|x^h(\tau) - w^0(\tau)\right|_N^2 \right\} d\tau. \tag{3.15}$$

In virtue of (3.14), (3.15), valid is the estimate

$$\lambda_h(t) \leqslant k_5\left(h + \omega(\delta) + \delta + \int\limits_{t_0}^{t} \Big\{|s^h(\tau) - x^h(\tau)|_N^2 + \lambda_h(\tau)\Big\}\, d\tau\right). \tag{3.16}$$

Using Lemma 2, for sufficiently small h ($h \in (0, h_1)$) we establish from (3.16) that

$$\lambda_h(t) \leqslant k_6\left(h + \omega(\delta) + \delta + (h + \omega_2(\delta) + \delta + \alpha)^{1/2} + (h + \omega_2(\delta) + \delta)\alpha^{-1} + \int\limits_{t_0}^{t} \lambda_h(\tau) d\tau\right).$$

Then, in virtue of the Gronwall lemma [6], we obtain

$$\lambda_h(t) \leqslant k_7\Big\{(h + \omega_2(\delta) + \delta + \alpha)^{1/2} + (h + \omega_2(\delta) + \delta)\alpha^{-1} + \omega(\delta)\Big\}, \quad t \in T.$$

It follows from the last inequality that the family $(\Delta_h, S_h, U_h)_{h>0}$ of y-procedures of control where S_h are determined according to (3.4) and U_h satisfy (3.10) guarantees stable guidance, which proves assertion (2) of Theorem 2.

4. CONCLUSIONS

The paper considered the problem of control of a system of ordinary differential equations under the assumption that along with the control the system is subjected to an uncontrollable perturbation. An algorithm for solution of the problem in the case of incomplete information about the phase trajectory (measurement of part of coordinates) is given which is stable to the information noise and errors of calculation of the coordinates.

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